

bear out the inferences which Messrs. Campbell and Neison have made from them. The authors appear to have overlooked the fact that if personal systematic differences exist in the observations of the limbs, it is only the unequal distribution amongst the observers of the observed diameters and observations at Quadratures which affect the final result.

*On the Inclination of the Ring of Saturn to its Orbit, deduced from Washington Observations.* By Professor Edward S. Holden.

(Communicated by the Secretaries.)

The observations given in the following pages were made by Professor A. Hall and myself during the years 1877, 1878, and 1879, which were favourable for the purpose.

The instruments employed were the 26-inch Clark Refractor, cirtomethe filar mier, and eyepieces magnifying from 400 to 800 times. The resulting position-angles of the major axis of the ring are given in the fifth column of the table. The reductions were made by the following formulæ, where  $p$  is the angle of the northern half of the minor axis of the Ring and the Declination-circle through *Saturn's* centre.

The formulæ are Bessel's, and are given in Engelmann's Bessel's *Abhandlungen*, vol. i. p. 321.

$$\begin{aligned}\cos a \cdot \cos A &= \cos p, \\ \cos a \cdot \sin A &= \sin p \cdot \sin \delta, \\ \sin a &= \sin p \cdot \cos \delta, \\ \tan B &= \frac{\tan a}{\cos (n - a + A)}, \\ \tan C &= \tan (n - a + A) \frac{\cos B}{\cos (B + i)}, \\ \tan i' &= \tan (B + i) \frac{\cos C}{\cos (n'' - n' + C)}.\end{aligned}$$

In these formulæ  $a$ ,  $\delta$ , are the geocentric coordinates of *Saturn*;  $i$  is the inclination of *Saturn's* orbit to the Earth's equator, and  $n$  the longitude of the node of *Saturn's* orbit on the same plane;  $n'$  is the node of the Earth's equator on *Saturn's* orbit, and  $n''$  is the node of the Ring-plane on *Saturn's* orbit,  $i'$  being the (required) inclination of these planes.

I have assumed Bessel's values of  $n$ ,  $n'$ ,  $n''$ , and  $i$ , since the introduction of Le Verrier's or Hill's values for  $\varpi$  and  $i$  will not change the results for  $i'$ .

The following table gives each observation and the resulting value of  $i'$ .

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of the Ring of Saturn.

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Table giving Washington Observations of Saturn's Ring, and the Computed Values of the Inclinations of the Plane of the Ring to Saturn's Orbit.

No.	Date.	Date.	Wash. M.T.	p		Obs.	Hall. i' =			Holden. i' =		
				h	m		°	'	"	°	'	"
1	June 18	1877.46	15 8	94.82	H.		26	42	35			
2	19	.47	15 20	4.80	H.		26	44	39			
3	22	.47	14 50	5.18	H.		27	4	34			
4	23	.48	14 57	4.76	H.		26	39	23			
5	25	.49	14 46	4.80	H.		26	41	55			
6	July 10	.52	14 50	5.20	Hn.					27	5	14
7	11	.53	14 6	4.68	Hn.					26	40	52
8	11	.53	14 7	4.58	Hn.					26	28	27
9	13	.53	14 0	5.03	Hn.					26	55	19
10	Aug. 3	.60	12 13	5.3	Hn.					27	7	43
11	10	.61	13 38	4.75	H.		26	32	49			
12	11	.61	11 36	4.73	H.		26	59	1			
13	15	.63	12 35	5.01	H.		26	46	55			
14	16	.63	12 54	4.98	H.		26	44	46			
15	17	.63	13 8	4.93	H.		26	41	24			
16	18	.64	12 12	4.98	H.		26	44	10			
17	27	.66	13 34	4.93	H.		26	38	0			
18	28	.66	13 13	4.78	H.		26	28	40			
19	Sept. 23	.73	10 44	5.13	H.		26	40	12			
20	29	.75	9 40	5.08	H.		26	35	9			
21	Oct. 1	.75	10 9	5.18	H.		26	40	36			
22	2	.75	10 28	5.00	H.		26	29	22			
23	5	.76	7 17	5.27	Hn.					26	44	46
24	9	.78	9 24	5.75	Hn.					27	12	30
25	13	.79	10 47	5.77	Hn.					27	11	35
26	13	.79	9 19	5.18	H.		27	6	40			
27	14	.79	8 25	5.30	H.		26	47	38			
28	14	.79	9 59	5.56	Hn.					26	58	44
29	15	.79	11 12	5.68	Hn.					27	6	45
30	16	.79	10 39	5.71	Hn.					27	8	20
31	17	.80	7 20	5.09	H.		26	30	40			
32	18	.80	6 56	5.26	H.		26	40	40			
33	18	.80	9 49	5.66	Hn.					27	4	49
34	23	.81	7 5	5.21	H.		26	36	33			
35	24	.81	7 35	5.29	H.		26	41	25			

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No.	Date.	Date.	Wash. M.T.	$p$	Obs.	Hall. $i' =$	Holden. $i' =$
			h m	$^{\circ}$		$^{\circ}$ ' "	$^{\circ}$ ' "
36	Dec. 2	1877'93	6 26	5'10	H.	26 28 38	0 ' "
37	7	'94	6 48	5'40	H.	26 47 18	
38	8	'94	6 9	5'15	H.	26 32 30	
39	9	'94	6 18	5'27	H.	26 39 56	
40	15	'96	7 2	4'80	H.	26 12 56	
41	19	'97	6 52	5'20	H.	26 37 56	
42	31	'99	6 23	5'55	H.	27 2 44	
43	Jan. 3	1878'01	6 12	5'50	H.	27 0 44	
44	5	'01	6 18	4'95	H.	26 28 27	
45	12	'03	6 4	5'17	H.	26 44 30	
46	16	'04	6 6	4'90	H.	26 30 2	
47	17	'05	6 10	4'97	H.	26 34 40	
48	19	'05	5 51	4'83	H.	26 27 10	
49	Sept. 9	1878'69	10 30	4'5	Hn.		27 15 52
50	17	'71	10 40	4'1	Hn.		26 52 41
51	19	'72	10 50	3'9	Hn.		26 40 55
52	Oct. 1	'75	11 30	4'5	Hn.		27 11 8
53	8	'77	11 0	4'4	Hn.		26 59 19
54	14	'79	9 30	3'9	Hn.		26 29 14
55	21	'81	8 30	4'6	Hn.		27 6 7
56	25	'82	9 18	4'5	Hn.		26 59 56
57	26	'82	9 37	4'2	Hn.		26 41 55
58	Nov. 5	'85	10 44	4'5	Hn.		26 59 35
59	13	'87	8 33	3'60	H.	26 12 52	
60	13	'87	9 42	4'2	Hn.		26 36 56
61	14	'87	7 50	3'90	H.	26 18 50	
62	14	'87	8 53	4'3	Hn.		26 42 55
63	15	'88	7 30	3'85	H.	26 15 45	
64	18	'89	9 13	4'23	H.	26 38 29	
65	20	'89	8 23	4'2	Hn.		26 36 41
66	Oct. 16	1879'80	11 5	3'4	Hn.		27 11 30
67	20	'81	9 48	3'3	Hn.		27 8 34
68	23	'81	12 0	3'2	Hn.		27 4 40
69	24	'82	8 48	3'3	Hn.		27 11 35
70	25	'82	9 47	3'2	Hn.		27 6 7
71	26	'82	8 38	3'4	Hn.		27 19 15
72	27	'82	12 5	3'2	Hn.		27 7 45
73	28	'83	10 0	3'1	Hn.		27 2 11

No.	Date.	Date.	Wash. M.T.	<i>p</i>	Obs.	Hall. <i>i'</i> =	Holden. <i>i'</i> =
			h m	o		o ' "	o ' "
74	30	'83	10 6	3'3	Hn.		26 59 41
75	31	'83	10 21	3'1	Hn.		26 46 55
76	Nov. 11	'87	9 49	3'5	Hn.		27 18 40
77	22	1879'89	8 36	93'3	Hn.		26 55 2

From the 40 observations of Professor Hall we have for

$$1877.948 = 1877, \text{ Dec. 11,}$$

$$i' = 26^{\circ} 38' 47'' \pm 1'.4.$$

The probable error of a single observation is  $\pm 8'.6$ .

From the 37 observations made by me, we have for

$$1878.771 = 1878, \text{ Oct. 8,}$$

$$i' = 26^{\circ} 57' 2'' \pm 1'.7.$$

The probable error of a single observation is  $\pm 10'.3$ .

Bessel's 22 observations give for

$$1818.726 = 1818, \text{ Sept. 20,}$$

$$i' = 27^{\circ} 0' 9'' \pm 5'.2.$$

The probable error of a single observation was  $\pm 24'.1$ .

I have not combined the results obtained by Professor Hall and myself, since the observations of 1877, October 13 and 14, and of 1878, Nov. 13 and 14, show that they are not strictly comparable. My result agrees with Bessel's within the limits of the probable errors. A comparison of the results of Professor Hall and of Bessel would indicate a diminution of the inclination since 1818.

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*University of Wisconsin, Madison:*  
*1882, January.*

*Postscript.*—Since submitting the above to Professor Hall, I have found two observations of the position-angle of *Saturn's* Ring made by Dr. Auwers in 1861 with Königsberg Heliumeter in the publications of that Observatory for 1865, page 132. They are

$$1861, \text{ April 15; } p = 83^{\circ} 14' (2),$$

$$23; \quad = 82^{\circ} 39' (2).$$

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